Systems. I: Open-Loop On-Line Optimizing Control," *AIChE J.*, **27**, 1,960 (1981).

Gill, P. E., W. Murray, and M. H. Wright, *Practical Optimization*, Academic Press, New York (1981).

Govind, R., and G. Powers, "Control System Synthesis Strategies," *AIChE J.*, **28**, 60 (1982).

Grossmann, I. E., K. P. Halemane, and R. E. Swaney, "Optimization Strategies for Flexible Chemical Processes," *Proc. Int. Symp. Proc. Sys. Eng.*, Kyoto (1982).

Han, C. D., "Determination of Start-Up Conditions for Chemical Reactor Stability," *Ind. Eng. Chem. Fund.*, **9**, 634 (1970).

Himmelblau, D. M., *Applied Nonlinear Programming*, McGraw-Hill, New York (1972).

Jensen, N. D., D. G. Fisher, and S. L. Shah, "Interaction Analysis in Multivariable Control System Design," to appear in *AIChE J.*

Kao, Y. K., "A Simple Start-Up Policy for a Catalytic Reactor with Catalyst Deactivation," *Chem. Eng. J.*, **20**, 237 (1980).

Lau, H., "Studies on the Control of Integrated Chemical Processes," Ph.D. Thesis, U. Minnesota (1982).

Lau, H., J. Alvarez, and K. F. Jensen, "Synthesis of Control Structures by Singular Value Analysis: Dynamic Measures of Sensitivity and Interaction," *AIChE J.* (Mar., 1985).

Lee, W., and W. V. Weekman, Jr., "Advanced Control Practice in the Chemical Process Industry: A View from Industry," *AIChE J.*, **22**, 27 (1976).

Lenhoff, A. M., and M. Morari, "Design of Resilient Processing Plants. I: Process Design Under Considerations of Dynamic Aspects," *Chem. Eng. Sci.*, **37**, 245 (1982).

Marselle, D. F., M. Morari, and D. F. Rudd, "Design of Resilient Processing Plants. II: Design and Control of Management Systems," *Chem. Eng. Sci.*, **37**, 259 (1982).

Morari, M., "Studies in the Synthesis of Control Structures of Chemical Processes," Ph.D. Thesis, U. Minnesota (1977).

Morari, M., Y. Arkun, and G. Stephanopoulos, "Studies in the Synthesis of Control Structures for Chemical Processes. I: Formation of the Problem. Process Decomposition and the Classification of the Control Tasks. Analysis of the Optimizing Control Structures," *AIChE J.*, **26**, 220 (1980).

Morari, M., and G. Stephanopoulos, "Studies in the Synthesis of Control Structures for Chemical Processes. II: Structural Aspects and the Synthesis of Alternative Feasible Control Schemes," *AIChE J.*, **26**, 232 (1980).

Morari, M., "Flexibility and Resiliency of Process Systems," *Proc. Int. Symp. Proc. Sys. Eng.*, 223, Kyoto (1982).

Morari, M., "Design of Resilient Processing Plant. IV," *Chem. Eng. Sci.* (1983).

Morari, M., J. Oglesby, and J. D. Prosser, "Design of Resilient Processing Plants. V," *Chem. Eng. Sci.* (1983).

Noble, B., and J. W. Daniel, *Applied Linar Algebra*, 2nd Ed., Prentice Hall, Englewood Cliffs, NJ (1977).

Prett, D. M., and R. D. Gillette, "Optimization and Constrained Multivariable Control of a Catalytic Cracking Unit," *Proc. JACC*, San Francisco (1980).

Ray, W. H., and S. P. Hastings, "The Influence of Lewis Number on the Dynamics of Chemically Reacting Systems," *Chem. Eng. Sci.*, **35**, 589 (1980).

Ray, W. H., "Multivariable Process Control—A Survey," *Proc. Int. Symp. Proc. Sys. Eng.*, Kyoto (1982).

Romagnoli, J., J. Alvarez, and G. Stephanopoulos, "Variable Control Structure," *Proc. JACC*, San Francisco (1980).

Rosenbrock, H. H., *State Space and Multivariable Theory*, Wiley-Interscience, New York (1970).

Stephanopoulos, G., "Synthesis of Control Systems for Chemical Plants—A Challenge for Creativity," *Proc. Int. Symp. Proc. Sys. Eng.*, 131, Kyoto (1982).

Takamatsu, T., "The Nature and Role of Process Systems Engineering," *Proc. Int. Symp. Proc. Sys. Eng.*, 3, Kyoto (1982).

Tomlin, J. A., "On Scaling Linear Programming Problems," *Math. Program. Study*, **4**, 146 (1975).

Tung, L. S., and T. F. Edgar, "Dynamic Interaction Analysis and Its Application to Distillation Column Control," *IEEE Decision and Control Conf.*, 1,107 (1977).

Webb, P. U., "Successful Optimizing Control Systems in Process Operations," *Proc. JACC*, San Francisco (1980).

Weber, R., and C. Brosilow, "The Use of Secondary Measurements to Improve Control," *AIChE J.*, **18**, 614 (1972).

Witcher, M. F., and T. J. McAvoy, "Interacting Control Systems: Steady State and Dynamic Measurement of Interaction," *ISA Trans.*, **26**(3), 35 (1977).

Manuscript received February 18, 1983; revision received November 22 and accepted December 6, 1983.

R & D NOTES

Sedimentation and Fluidization in Solid-Liquid Systems: A Simple Approach

V. S. PATWARDHAN
and CHI TIEN

Department of Chemical Engineering and
Materials Science
Syracuse University
Syracuse, NY 13210

Because of the interactions among suspension particles, the settling velocity of particles in suspensions is slower than the terminal velocity of a single particle of the same size. The most accurate empirical expression relating the settling velocity to the suspension porosity is represented by

$$U_R = \epsilon^n \tag{1}$$

where U_R is the ratio of the particle velocity relative to the liquid during settling to the terminal velocity of a particle in an infinite medium. (Garside and AlDibouni, 1977) The exponent, n, depends only upon the Reynolds number, (N_{Re}), which is based on the terminal velocity. This expression is similar to the correlation proposed by Richardson and Zaki (1954), although their values of n were slightly different from those of Garside and AlDibouni.

V. S. Patwardhan is on leave from Chemical Engineering Div., National Chemical Laboratory, Pune 411 008, India.

A different approach was taken by Ishii and Zuber (1979). Ishii and Zuber defined a Reynolds number and a drag coefficient for suspension particles using the relative velocity of the settling particles with respect to the liquid as

$$N_{Re} = d\rho_\ell v_r/\mu_m \qquad (2)$$

and

$$C_D = 4d(\rho_s - \rho_\ell)g\epsilon/(3\rho_\ell v_r^2) \qquad (3)$$

These investigators proposed that the relationship between C_D and N_{Re} for particles in suspensions is identical to the well-known C_D-N_{Re} relationship for a single particle at low N_{Re}. The $U_R - \epsilon$ relationship obtained on this basis at low Reynolds numbers, was shown to compare well with experimental data.

Equation 3 also predicts that if $\rho_s > \rho_\ell$, solid particles should always move downwards in the suspending liquid. This prediction is contrary to experimental observations reported by Richardson and Meikle (1961), who found that particles which are heavier than liquid may actually move upward in the liquid if the suspension density is high enough.

Masliah (1979) presented an equation for hindered settling of polydispersed particles in suspensions at low N_{Re}; that equation may be written as

$$U_{Ri} = F(\epsilon)(\rho_{si} - \rho_m)/(\rho_{si} - \rho_\ell) \qquad (4)$$

Masliah's treatment explains the observations of Richardson and Meikle (1961) but incorporates an empirical porosity function $F(\epsilon)$ and is not applicable to high N_{Re}.

Here, a new method is proposed for predicting the U_R-ϵ relationship of particles in suspensions at all N_{Re}. Predictions based on new method are shown to compare well both with experimental data and with earlier correlations.

THEORY

The relationship between the drag coefficient and the N_{Re} (based on the terminal velocity) for a single particle settling in a liquid is available in literature (e.g., Leva, 1959). The calculation of the terminal velocity is facilitated by introducing a new parameter,

$$\alpha = C_D(N_{Re,t})^2 = 4gd^3\rho_\ell(\rho_s - \rho_\ell)/(3\mu_\ell^2) \qquad (5)$$

which is independent of the terminal velocity. The N_{Re} (based on the terminal velocity) is defined as

$$N_{Re,t} = U_t d\rho_\ell/\mu_\ell \qquad (6)$$

and may be expressed in terms of α as

$$N_{Re,t} = \beta(\alpha) = A\alpha^B\{(1 - F)[1 - \exp(-C_1\alpha^{D_1})]^{E_1} + F[\tanh(C_2\alpha^{D_2})]^{E_2}\} \qquad (7)$$

where $A = 1.5076$, $B = 0.5$, $C_1 = 0.1811$, $D_1 = 0.2381$, $E_1 = 2.1$, $C_2 = 0.3480$, $D_2 = 0.1471$, $E_2 = 3.4$, and $F = 3.9$. The predictions of Eq. 7 agree within 5% with the $N_{Re,t} - \alpha$ relationship given by Wallis (1969). It is proposed here that this relationship also holds true for particles in suspensions for which the corresponding $N_{Re,m}$ and α_m are defined for suspensions, or

$$\alpha_m = 4gd^3\rho_m(\rho_s - \rho_m)/(3\mu_m^2) \qquad (8)$$

and

$$N_{Re,m} = v_r d\rho_m/\mu_m \qquad (9)$$

where the subscript m refers to the properties of the suspension. For a suspension of particles of the same density, ρ_m is given by

$$\rho_m = \rho_s(1 - \epsilon) + \rho_\ell\epsilon \qquad (10)$$

The suspension viscosity is related to the porosity of the suspension (i.e., volume concentration) (Thomas, 1965) as

$$M = \mu_m/\mu_\ell = 1 + 2.5(1 - \epsilon) + 10.05(1 - \epsilon)^2 + 0.00273\exp[16.6(1 - \epsilon)] \qquad (11)$$

The term v_r of Eq. 9 represents the relative velocity between the suspended particle and the liquid medium. It is equal to U_o/ϵ in the case of batch fluidization and to U_t in the case of a single particle settling in an infinite liquid. Using Eq. 5 through 9, the following relationships can be easily established:

$$\alpha_m = \alpha\epsilon[\epsilon + (1 - \epsilon)R]/M^2 \qquad (12)$$

where R represents the density ratio ρ_s/ρ_ℓ, and

$$N_{Re,m} = N_{Re,t}U_R[\epsilon + (1 - \epsilon)R]/M \qquad (13a)$$

or

$$U_R = M\left(\frac{N_{Re,m}}{N_{Re,t}}\right)\Big/[\epsilon + (1 - \epsilon)R] \qquad (13b)$$

From Eq. 7 and the hypothesis that the $N_{Re,t} - \alpha$ relationship is the same as the $N_{Re,m} - \alpha_m$ relationship, the above expression becomes

$$U_R = M\beta(\alpha_m)/\{[\epsilon + (1 - \epsilon)R]\beta(\alpha)\} \qquad (14)$$

where $\beta(\alpha_m)$ is of the same functional form of $\beta(\alpha)$ (i.e., Eq. 7) with α_m replacing α. Thus, U_R depends upon α (which defines the regime according to Eq. 5); ϵ (i.e., the suspension concentration); and R, the density ratio. For two limiting cases of high and low $N_{Re,t}$, the following expressions are obtained:

(i) *Low* $N_{Re,t}$. This refers to the Stokes regime, where $\beta(\alpha) = \alpha/24$. Eq. 14 gives

$$U_R = \epsilon/M \qquad (15)$$

(ii) *High* $N_{Re,t}$. This refers to the Newtons regime where $\beta(\alpha) = (\alpha/0.44)^{0.5}$. Equation 14 gives

$$U_R = \{\epsilon/[\epsilon + (1 - \epsilon)R\}^{0.5} \qquad (16)$$

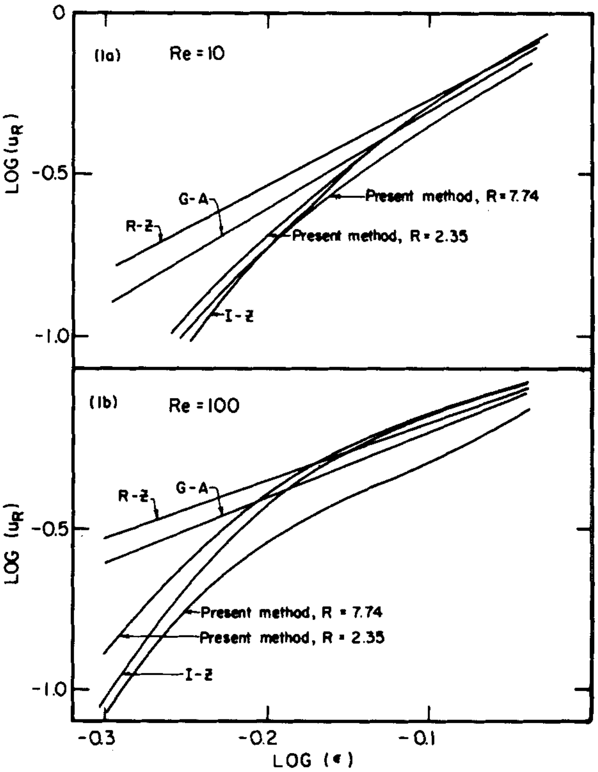

Figure 1. Comparison of the present method with earlier correlations.

R–Z: Richardson and Zaki (1954)

G–A: Garside and AlDibouni (1977)

I–Z: Ishii and Zuber (1979)

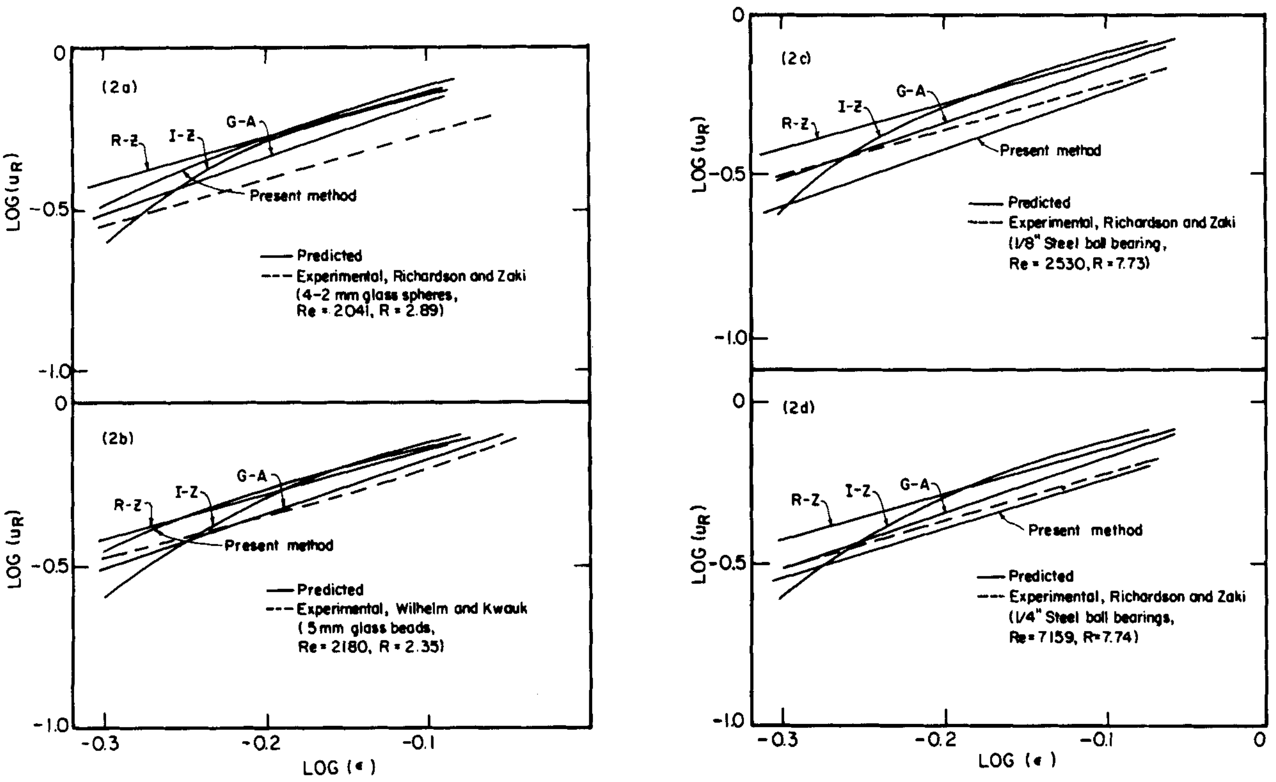

Figure 2. Comparison of the present method and earlier correlations with experimental data in the Newton's regime.

COMPARISON WITH EXPERIMENTAL DATA

(i) *Low* $N_{Re,t}$. The U_R-ϵ relationship predicted by the present method at low $N_{Re,t}$ is given by Eq. 15, which is identical to the expression proposed by Ishii and Zuber earlier (1979). As pointed out by these authors, Eq. 15 agrees well with experimental data at low $N_{Re,t}$.

(ii) *Intermediate* $N_{Re,t}$. Figure 1 shows the predictions of the present method and those of the earlier correlations at two different values of $N_{Re,t}$. The earlier correlations give a unique U_R-ϵ relationship at a given $N_{Re,t}$. The present method, however, includes another parameter, i.e., the density ratio, R, as evidenced by Eq. 14. In the Stokes' law regime at low $N_{Re,t}$, viscous forces dominate, and R is unimportant (Eq. 15). As $N_{Re,t}$ increases, however, inertia becomes more and more important, and so does R. Figure 1 shows two lines predicted by the present method for different values of R (corresponding approximately to the fluidization of glass and steel spheres by water). The difference between these lines is not very large, and both are seen to agree quite well with the method of Ishii and Zuber. Both the present method and that of Ishii and Zuber predict lower U_R at low ϵ than the correlations of Richardson and Zaki (1954) and Garside and AlDibouni (1977). The original data of Richardson and Zaki also indicate this trend, however, as Ishii and Zuber have pointed out.

(iii) *Large* $N_{Re,t}$. The experimental data reported by Richardson and Zaki (1954) and by Wilhelm and Kwauk (1948) which fall into this regime are shown in Figure 2 and cover the range of $R = 2.35$ to 7.74. The broken lines were drawn through the experimental data. The predictions of the present method are seen to agree well with the lines representing experimental data. The figure also shows that the predictions of the present method compare very well with those of the earlier correlations. The present method differs qualitatively from the earlier ones in one respect, i.e., it predicts that the U_R-ϵ relationship depends upon R. This dependence is most pronounced at high $N_{Re,t}$. The amount of experimental data available in this regime is, however, insufficient to conclude whether or not such a dependence is observed in practice.

One of the advantages of the present method is that it leads to a single equation, Eq. 14, which is applicable at all $N_{Re,t}$ and reduces to the appropriate asymptotic forms. There is no need to divide the complete range of $N_{Re,t}$ into different regimes.

The present method can be applied easily to systems involving particles of different types. During sedimentation and fluidization of such systems, the different types of particles also tend to segregate. The application of the present method to such a system at low $N_{Re,t}$ gives

$$U_{Ri} = \left(\frac{\rho_{si} - \rho_m}{\rho_{si} - \rho_\ell}\right)\Big/ M \qquad (17)$$

where the subscript i refers to a particular type of particle. The $(\rho_{si} - \rho_m)$ term in the numerator implies that particles heavier than the liquid can move upwards with respect to the liquid if ρ_m is sufficiently high. This equation is identical to Eq. 4, which is based on the derivative presented by Masliah (1979), except for the fact that $F(\epsilon)$ is replaced by $1/M$. Masliah applied his equation to the experimental data of Richardson and Meikle (1961), which covered the range of $\epsilon = 0.7$ to 1.0. It can be shown rather easily that $F(\epsilon)$ and $1/M$ are very nearly equal in this range of ϵ.

CONCLUSIONS

A new method is developed for predicting the velocity-voidage relationship for sedimentation and fluidization of solid-liquid systems at all Reynolds numbers. It is based on the use of the C_D-N_{Re} relationship for a single particle and the use of suspension density and suspension viscosity. The predictions are shown to be in good agreement with experimental data and earlier correlations. The method is also applicable to systems involving several species of particles.

ACKNOWLEDGMENT

This study was performed under Grant CPE 7908893, National Science Foundation.

NOTATION

C_D = drag coefficient
d = particle diameter
$F(\epsilon)$ = a function appearing in Eq. 4
g = acceleration resulting from gravity
M = μ_m/μ_ℓ
N_{Re} = Reynolds number defined by Eq. 2
$N_{Re,m}$ = Reynolds number for a suspension defined by Eq. 9
$N_{Re,t}$ = Reynolds number based on terminal velocity, defined by Eq. 6
R = ρ_s/ρ_ℓ
U_o = superficial liquid velocity
U_R = v_r/U_t
U_t = terminal velocity of a particle in an infinite medium
v_r = relative velocity between liquid and particles

Subscripts

i = ith particle species
ℓ = liquid
m = mixture or suspension
s = solid

Greek Letters

α = $C_D(N_{Re,t})^2$
β = functional form in Eq. 7
ϵ = porosity
ρ = density
μ = viscosity

LITERATURE CITED

Garside, J., and M. R. AlDibouni, "Velocity-Voidage Relationships for Fluidization and Sedimentation in Solid Liquid Systems," *Ind. Eng. Chem. Proc. Des. Dev.*, **16**, 206 (1977).

Ishii, M., and N. Zuber, "Drag Coefficient and Relative Velocity in Bubbly, Droplet or Particulate Flows," *AIChE J.*, **25**, 843 (1979).

Leva, M., *Fluidization*, McGraw-Hill, New York (1959).

Masliah, J. H., "Hindered Settling in a Multi-species Particle System," *Chem. Eng. Sci.*, **34**, 1166 (1979).

Richardson, J. F., and R. A. Meikle, "Sedimentation of Uniform Fine Particles and of Two Component Mixtures of Solids," *Trans. Inst. Chem. Engrs.*, **39**, 348 (1961).

Richardson, J. F., and W. N. Zaki, "Sedimentation and Fluidization," *Trans. Inst. Chem. Engrs.*, **32**, 35 (1954).

Thomas, D. G., "Transport Characteristics of Suspensions: A Note on the Viscosity of Newtonian Suspensions of Uniform Spherical Particles," *J. Colloid. Sci.*, **20**, 267 (1965).

Wallis, G. B., *One Dimensional Two Phase Flow*, McGraw Hill, New York (1969).

Wilhelm, R. H., and M. Kwauk, "Fluidization of Solid Particles," *Chem. Eng. Prog.*, **44**, 201 (1948).

Manuscript received October 7, 1982; revision received June 3, and accepted June 23, 1983.

Electroconvective Enhancement of Mass or Heat Exchange Between a Drop or Bubble and Surroundings in the Presence of an Interfacial Tension Gradient

L. S. CHANG and J. C. BERG

Department of Chemical Engineering BF-10
University of Washington
Seattle, WA 98195

Experimental evidence suggests (Thornton, 1968; Bailes and Thornton, 1971; Harker and Ahmadzadeh, 1974; Bailes, 1981) and computations appear to confirm (Morrison, 1977; Griffiths and Morrison, 1979; Chang et al., 1982; Chang and Berg, 1983) that electric fields may, under certain circumstances, lead to significant enhancements in the mass (or heat) exchange rate between drops or bubbles and a surrounding liquid medium. The premise of the computational models is that such enhancement may be traced to electric-field-induced circulation in the drop, as first described by G. I. Taylor (1966). When the drop is stationary with respect to the external medium and the electric field is uniform, the circulation takes the form of a symmetrical pair of tori inside the drop, with surface flow directed either from the poles toward the equator or vice versa, depending on the electrical properties of the system. For the situation more likely to exist in practice, in which the drop or bubble translates by gravity and the electric field is oriented parallel to the translation, the flow becomes a hybrid between the single torus, Hadamard-Rybczynski circulation and the electrohydrodynamic circulation, as described by Taylor (Chang et al., 1982; Chang and Berg, 1983). The torus on the rear side of the drop is reduced in size and may be absent altogether. Under the latter conditions, no significant electroconvective enhancement of exchange rates may be anticipated. The extent of flow modification by the electric field may be conveniently characterized in terms of the dimensionless parameter W (Chang et al., 1982):

$$W = \frac{4V(1 + X)}{U_0} \tag{1}$$

where V is the maximum electrically-induced interfacial velocity in the corresponding Taylor problem and is given by:

$$V = \frac{9E^2 a \epsilon_2 (1 - \epsilon_1 S_1/\epsilon_2 S_2)}{10(2 + S_1/S_2)^2 \mu_1 (1 + X)} \tag{2}$$

L. S. Chang is presently with the Union Carbide Corp.
Correspondence concerning this paper should be addressed to J. C. Berg.